Residential Demand Response for Renewable Energy Resources in Smart Grid Systems

Laihyuk Park, Yongwoon Jang, Sungrae Cho, and Joongheon Kim

***Abstract*—With the current state of development in demand response (DR) programs in smart grid systems, there have been great demands for automated energy scheduling for residential customers. Recently, energy scheduling in smart grids have focused on the minimization of electricity bills, the reduction of the peak demand, and the maximization of user convenience. Thus, a user convenience model is proposed under the consideration of user waiting times, which is a non-convex problem. Therefore, the non-convex is reformulated as convex to guarantee optimal solutions. Moreover, mathematical formulations for DR optimization are derived based on the reformulated convex problem. In addition, two types of pricing policies for electricity bills are designed in the mathematical formulations, i.e., real-time pricing policy and progressive policy. With real-time pricing policy, convexity is guaranteed whereas progressive policy cannot. Then, heuristic algorithms are finally designed for obtaining approximated optimal solutions in progressive policy.**

***Index Terms*—Smart Grid, Residential Energy Resources, Convex Optimization, Demand Response**

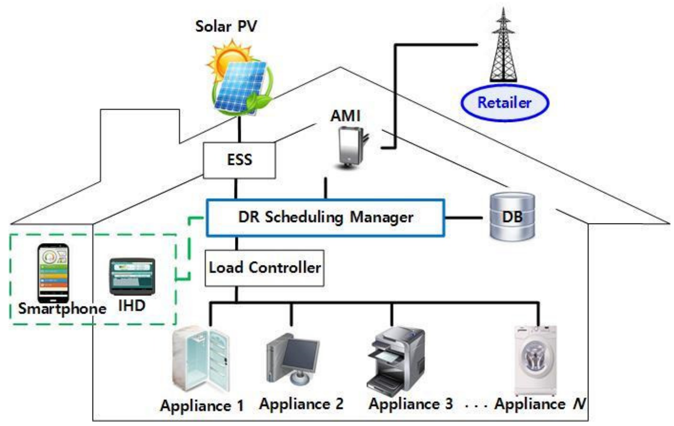

Fig. 1: The concept of smart grid residential system architecture

I. Introduction

In the last few decades, global electricity consumption has dramatically increased and fluctuated in uncertain ways, causing blackouts. Due to the unexpected peak electricity demands, a significant electricity supply is required. One promising solution to this problem is the use of smart grid systems envisioned as a future power system [1]–[3]. The smart grid systems are capable to reduce the electricity peak and induce effective electricity consumption through various price policies, demand response (DR) control methodologies, and state-of-the-art smart equipment in order to optimize electricity resource usage in an efficient way [4]–[6]. With the current state of development in smart grid systems, there in a strong demand for automated scheduling schemes on the consumer side in residential smart grid systems. Fig. 1 shows the up-to-date concept of residential smart grid systems. As shown in the Fig. 1, the overall composition of the system contains a retailer, advanced metering infrastructure (AMI), load controller, scheduling manager, database, and a number of appliances. DR is one of the key technologies for smart grid systems and there are two types of DR in the literature, i.e., *incentive-based* programs and *price-based* programs [7]. The incentive-based DR program is designed to induce smaller amount of electricity use at times of high market price or when grid reliability is "jeopardized". On the other hand, a price-based DR program is defined as a tariff or program established to motivate changes in the price of electricity over time. In the price-based DR programs, the key function is for scheduling the activation time of the requested load, which can be shifted to reduce the electricity bill and peak consumption. This paper focuses on price-based DR programs because it is the dominant technology in the literature at this moment. For the price-based DR programs, the utilities can change the power consumption of customers via pricing, such as time of use (TOU), critical peak pricing (CPP), extreme day CPP (ED-CPP), extreme day pricing (EDP), and real-time pricing (RTP). In these pricing policies, consumer real time pricing information comes from the power grid retailer or utility. In reactive pricing, *real time pricing* is formulated via linear programming. The concept of *day ahead pricing* is introduced in [8] which receives real-time pricing information from the power grid retailer or utility. However, many commercial retailers such as the Korea Electric Power Corporation (KEPCO) have yet to support real time pricing. Instead of the use of real time pricing, they follow a *progressive scheme* (also called *progressive pricing*). In the progressive scheme, the more one consumes electricity, the more one pays per power unit (KWH). In the literature, the varieties of DR programs [9], [10] have assumed the progressive scheme. The progressive pricing is formulated via non-linear programming. In order to solve this problem, [9] uses convex programming tools such as the interior point method (IPM), and [10] designed a dynamic programming-based solver.

Besides the electricity bill and peak consumption, recent studies have additionally considered user convenience models. Since there are various types of appliances, recent studies considered simplified and limited models for appliances in

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order to formulate DR problems as relatively easy problems (i.e., linear programming (LP) or convex problems).

In [8], the authors relaxed the conditions of convex optimization problems for cost minimization as well as user convenience maximization, which can be applied to a large number of consumers. They formulated the DR problem as an mixed integer non linear programming (MINLP) problem, which is hard to solve in a polynomial-time. Therefore, they approximated the problem as a convex problem by relaxing the integer variables to continuous variables. By solving the relaxed optimization problem, they obtained a suboptimal solution. Although the convex programming tools in [9] or the dynamic programming schemes in [10] can find approximate solutions, they cannot guarantee optimality.

In this paper, the convex programming framework to guarantee an optimal solution in real time pricing policy is proposed. As mentioned earlier, progressive schemes suffer from being NP-hard problems. However, our scheme presents a heuristic algorithm to solve NP-hard problems and obtain an approximate optimal solution for minimizing the electricity bill and peak consumption, as well as maximizing user convenience.

In summary, compared with previous contributions in the literature, the proposed schemes in this paper contributes the following.

- The proposed algorithm considers both the electricity cost and user convenience, which can be applied to solve the non-convex problem.
- The techniques that can convert a non-convex problem to a convex problem are proposed. By using these techniques, the proposed user convenience formula can be applied to solve the convex problem. Therefore, it is possible to guarantee an optimal solution in the real time pricing policy.
- For the progressive policy, a heuristic algorithm is proposed to obtain an approximated near-optimal solution for the given non-convex problem.

The rest of this paper is organized as follows. In Section II, the demand response program is proposed and the corresponding optimization problem is prsented. Section III presents a technique for converting the mathematical optimization framework of Section II to a convex form, which can guarantee optimal solutions for real time pricing policy. In addition, Section III also designs the heuristic algorithm which can obtain approximated optimal solutions for progressive policy. Section IV evaluates the performances and the conclusions of this paper are summarized in Section V.

II. System Model

This section formulates an optimization problem aims on the minimization of the total pricing while preserving user convenience. The purpose of this model is to shift the energy consumption schedules within residential appliances in order to optimize savings. To clarify, the purpose is not to reduce the amount of consumed energy but to find optimal energy schedules for each appliance that reduce the pricing and maximize convenience.

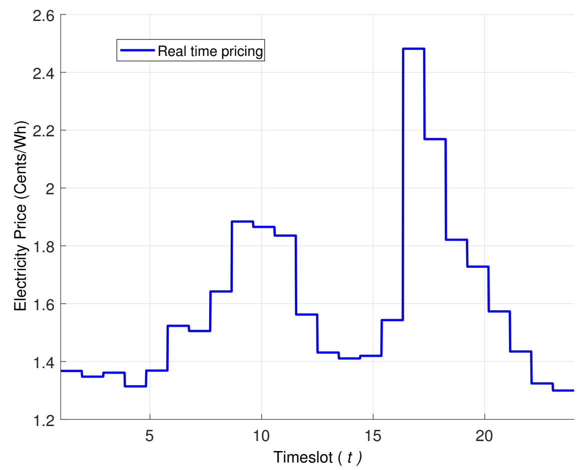

Fig. 2: Real time pricing example (The real-time prices are used by Illinois Power Company on 15 December 2009) [11] [12]

A. Energy Cost Model

In order to formulate required models, this paper defines the notations for the set of appliances $\mathcal{A}$, i.e.,

$$\mathcal{A} = \{a_1, a_2, a_3, a_4, \cdots, a_N\}, \tag{1}$$

where N is the total number of appliances in the smart grid system. This paper also assumes that the time is divided into equal timeslots t such that

$$t \in \mathcal{T} \text{ where } \mathcal{T} = \{1, 2, 3, \cdots, T\}. \tag{2}$$

The timeslot can be represented by any unit of time, and this paper assume that the unit time is an hour for simplicity. Based on the definitions and notations, the energy schedule of appliance i can be represented by

$$\mathcal{S}_i \triangleq \left[s_i^1, s_i^2, s_i^3, \cdots, s_i^t\right], \tag{3}$$

where

$$s_i^t = \begin{cases} 1, & \text{if appliance } a_i \text{ is operating at } t \\ 0, & \text{otherwise.} \end{cases}$$

This paper assumes that appliance a_i consumes energy E_i^{on} per each timeslot t during operation. Therefore, the energy consumption of all residential appliances per timeslot t (denoted as $E_{t,\mathcal{A}}$) can be modeled as follows:

$$E_{t,\mathcal{A}} \triangleq \sum_{a_i \in A} g_i \cdot s_i^t \cdot E_i^{on}, \tag{4}$$

where g_i indicates that appliance a_i is provided electricity from power grids or renewable energy resources. If appliance a_i is scheduled by using renewable energy resources, g_i is set to 0. Otherwise (e.g., it is scheduled by using an energy resource from the power retailer), it is set to 1.

In order to prevent blackout situations, energy consumption at time $t \in \mathcal{T}$ should be less than BO, i.e.,

$$E_{t,\mathcal{A}} \leq BO. \tag{5}$$

As mentioned earlier, two types of pricing models are considered in this paper, i.e., *real time pricing* and *progressive pricing*. Fig. 2 shows the real time pricing. The system with real time pricing assumes that the power retailer sends the day-ahead electricity pricing to the residential smart grid system. If the energy utility supports real time pricing, the cost function can be defined as follows:

$$B_t\left(E_{t,\mathcal{A}}\right) \triangleq E_{t,\mathcal{A}} \cdot p_t, \tag{6}$$

where p_t is the electricity price at time t. At this time, the cost function can be formulated with linear programming framework.

On the other hand, if the system assumes that the utility supports the progressive pricing, the cost function can be described as a quadratic cost function for the power generator in [9], which can be modeled as follows:

$$B_t\left(E_{t,\mathcal{A}}\right) \triangleq \alpha_t {E_{t,\mathcal{A}}}^2 + \beta_t E_{t,\mathcal{A}} + \gamma_t, \tag{7}$$

where $\alpha_t \geq 0$ and $\beta_t, \gamma_t \geq 0$ at each hour $t \in \mathcal{T}$.

Based on the assumptions and definitions, the price function $\mathcal{P}_t$ can be obtained as follows:

$$\mathcal{P}_t = B_t\left(\sum_{a_i \in A} g_i \cdot s_i^t \cdot E_i^{on}\right). \tag{8}$$

In order to model the renewable energy resources, several constraints are models as follows:

$$\begin{aligned} RER(t) = RER(0) + & \\ \int_{k=0}^{t-1} c(t) - \sum_{a \in \mathcal{A}} (1-g_i) \cdot s_i^t \cdot E_i^{on}, \forall t \in \mathcal{T}, & \end{aligned} \tag{9}$$

where $RER(t)$ is the expected available electricity power stored by energy storage systems (ESS) and $RER(0)$ is the ESS power from the beginning of the day. In (9), $c(t)$ is an expected charging amount from renewable energy resources. The considering smart grid system assumes that ESS has maximum battery level RER_{max}, and the constraint is formulated as follows:

$$0 \leq RER(t) \leq RER_{max}, \forall t \in \mathcal{T}. \tag{10}$$

B. User Convenience Model

The considering smart grid system assumes that a user can set their preferred begin operation time and end operation time for each appliance using a smartphone or IHD. In order to model the user convenience for a given appliance, the begin timeslot of appliance a_i and the end timeslot are denoted by C_i^{begin} and as C_i^{end}, respectively, where

$$C_i^{begin} < C_i^{end} \tag{11}$$

where $C_i^{begin} \in \mathcal{T}, C_i^{end} \in \mathcal{T}$. For example, the system assumes that the user wants the dishwasher to start after 13:00 and finish before 18:00. Then, the user must set the begin time and end time of the dishwasher as 13:00 and 18:00, respectively. In this case, $C_i^{begin} = 13$ and $C_i^{end} = 18$. Denoting by ρ_i which is the number of required timeslots for operating appliance a_i, ρ_i is less than or equal to $C_i^{begin} - C_i^{end}$. However, the operation of an appliance can be scheduled earlier or later than the user's setting. In this case, the user convenience decreases and dissatisfaction increases. Then the user convenience can be formulated using C_i^{begin} and C_i^{end}. Denoting w_i^t as the dissatisfaction degree of appliance a_i, w_i^t is given as follows:

$$w_t^i \triangleq \begin{cases} \kappa\left(C_i^{begin} - t\right), & t < C_i^{begin} \\ 0, & C_i^{begin} \leq t < C_i^{end} \\ \kappa\left(t - C_i^{end} + 1\right), & C_i^{end} \leq t, \end{cases} \tag{12}$$

where κ is a slope of dissatisfaction degree, which is used by the user to set the sensitivity with time.

Therefore, the user convenience function of appliance a_i can be expressed as follows where $t \in \{1, \ldots, T\}, T \triangleq |\mathcal{T}|$:

$$\mathcal{U}(a_i) = -\frac{\sum_{t \in \mathcal{T}} w_i^t \cdot s_i^t}{\rho_i}. \tag{13}$$

C. Objective Function

The optimization problems based on two objectives are formulated in the smart grid system where the two objective functions are for energy cost minimization and user convenience maximization, respectively.

The parameter λ is introduced as a weight factor to combine the energy cost model and user convenience model. For a fair comparison of two models, the scaling denominators, i.e., Γ_B and Γ_U are also introduced. The Γ_B is an expected maximum electricity bill per timeslot (i.e., all appliance are scheduled in one timeslot.) and the Γ_U is an expected maximum user convenience per timeslot (i.e., all appliance are scheduled to the farthest time slot in their preference time.), respectively. The objective function is obtained as follows with (5), (8), (9), (10),and (13):

$$\begin{aligned} \underset{s_i^t, g_i}{\text{minimize}} \sum_{t=1}^{T} \Bigg\{ & \frac{\lambda}{\Gamma_B} \cdot B_t\left(\sum_{a_i \in A} g_i \cdot s_i^t \cdot E_i^{on}\right) + \\ & \frac{(1-\lambda)}{\Gamma_U} \cdot \sum_{a_i \in \mathcal{A}} \frac{w_i^t \cdot s_i^t}{\rho_i} \Bigg\} \end{aligned} \tag{14}$$

subject to

$$\sum_{a_i \in A} s_i^t \cdot E_i^{on} \leq BO \tag{15}$$

$$0 \leq \lambda \leq 1 \tag{16}$$

$$\sum_{t}^{T} s_i^t = \rho_i \tag{17}$$

$$\begin{aligned} RER(t) &= RER(0) + \\ & \int_{k=0}^{t-1} c(k) - \sum_{a \in \mathcal{A}} (1-g_i) \cdot s_i^t \cdot E_i^{on} \end{aligned} \tag{18}$$

$$0 \leq RER(t) \leq RER_{max}, \forall t \in \mathcal{T}. \tag{19}$$

III. Proposed Optimization Framework

A. Re-Formulation: Convex Form

As shown in (12), w_i^t is a function of t. The formulation of user convenience (13) is non-convex since there exists the multiplication of a function w_i^t and a variable s_t^i [13]. Therefore, the defined objective function (16) is NP-complete and non-convex. For non-convex optimization problems, heuristic search algorithms such as genetic algorithm, simulated annealing, and tabu search can find approximated near-optimal solutions but the techniques cannot guarantee optimality. Therefore, the proposed objective function in previous section is re-formulated in order to eliminate non-convex terms.

In order to reformulate the given objective function, the parameter W^* is additionally introduced, which is a redundancy constraint. The defined parameter W^* is an arbitrary and sufficiently big number which is larger than all of the w_i^t. Therefore, it is obvious that the addition of following constraints is possible.

$$w_i^t \leq W^*, \tag{20}$$
$$0 \leq w_i^t. \tag{21}$$

Theorem 1: For the given non-convex formulation, introducing

$$w_i^t \leq W^* \cdot s_i^t \tag{22}$$

instead of (20) makes the formulation convex.

Proof 1: The proving procedure consists of two steps. The first step is for showing that (22) is equivalent to (20); and the next step is for showing that the proposed optimization framework with (22) is convex.

For the first step (i.e., showing that (22) is equivalent to (20)), we have to show that the two equations are equivalent when $s_i^t = 0$ and $s_i^t = 1$.

Suppose that $s_i^t = 1$. In this case, it is obvious that

$$w_i^t \leq W^* \cdot s_i^t\big|_{s_i^t=1}, \; (\Rightarrow) \; w_i^t \leq W^* \cdot 1, \tag{23}$$

by (22) and this equals to (20).

On the other hand, suppose that $s_i^t = 0$. For the non-convex formulation in previous section, $s_i^t = 0$ means that the appliance a_i is not scheduled at time t. Thus the corresponding dissatisfaction degree of appliance a_i at time t cannot exist, i.e., the optimization formulation in previous section will obviously return $w_i^t = 0$. The (22) with $s_i^t = 0$ can be as follows:

$$w_i^t \leq W^* \cdot s_i^t\big|_{s_i^t=0}, \; (\Rightarrow) \; w_i^t \leq W^* \cdot 0 = 0, \tag{24}$$

and thus $w_i^t = 0$ because w_i^t is definitely non-negative. As shown in the w_i^t values with (22) and (20) for each $s_i^t = 0$ and $s_i^t = 1$, it is clear that (22) is equivalent to (20). Finally, the first step is proved.

For the second step, it should be proved that the proposed optimization framework with (22) is convex. With (22), all $w_i^t \cdot s_i^t$ terms can be converted to w_i^t because they equals due to the first step as following Table I.

After this converting procedure, the all non-convex terms with the multiplication of two variables are eliminated, i.e., the terms contain only one linear variable w_i^t which is obviously convex [13]. Therefore, there are no non-convex terms in the proposed optimization formulation.

TABLE I: Elimination of Non-Convex Terms

	$w_i^t \cdot s_i^t$	w_i^t
$s_i^t = 0$	0	0 by (22)
$s_i^t = 1$	w_i^t	w_i^t by (22)

Based on the *[Theorem 1]*, (13) can be updated as follow:

$$\mathcal{U}^*(a_i) = -\frac{\sum_{t\in\mathcal{T}} w_i^t}{\rho_i}, \tag{25}$$

and thus this equation is now non-convex. Therefore, the final optimization framework will be as follows and note that this is convex:

$$\underset{s_i^t, g_i}{\text{minimize}} \sum_{t=1}^{T} \left\{ \frac{\lambda}{\Gamma_B} \cdot B_t \left(\sum_{a_i\in A} g_i \cdot s_i^t \cdot E_i^{on} \right) + \frac{1-\lambda}{\Gamma_U} \cdot \sum_{a_i\in\mathcal{A}} \frac{w_i^t}{\rho_i} \right\} \tag{26}$$

subject to

$$\sum_{a_i\in A} g_i \cdot s_i^t \cdot E_i^{on} \leq BO \tag{27}$$
$$0 \leq \lambda \leq 1 \tag{28}$$
$$w_i^t \leq W^* \cdot s_i^t \tag{29}$$
$$0 \leq w_i^t \tag{30}$$
$$\sum_{t}^{T} s_i^t = \rho_i \tag{31}$$

$$RER(t) = RER(0) + \int_{k=0}^{t-1} c(k) - \sum_{a\in\mathcal{A}} (1-g_i) \cdot s_i^t \cdot E_i^{on} \tag{32}$$
$$0 \leq RER(t) \leq RER_{max}, \forall t \in \mathcal{T}. \tag{33}$$

B. Heuristic Algorithm

As aforementioned, cost functions should be selected for either a progressive pricing (6) or real time pricing (7) for the energy utilities' policies. If the real time pricing is selected, the result can be obtained via the derived formula in section I. If the progressive pricing is selected, however, it is difficult to obtain the result since the cost function is not convex form. Therefore, a new novel heuristic algorithm should be designed for obtaining an approximated costs. The heuristic algorithm consists of appliance sorting and appliance scheduling. First, the sequence of appliance scheduling by appliance sorting should be determined. Second, we alternatively schedule each appliance by computing the objective function. The scheduling sequence is determined based on user convenience. For the heuristic algorithm formulation, the system assumes that user convenience of the appliance is more sensitive as the worst case for expected user convenience becomes larger. Therefore, the sensitivity of user convenience for appliance a_i (denote by SC_i) can be calculated as follows:

$$SC_i = \max(w_i^t), \forall t \in \mathcal{T}. \tag{34}$$

After sorting all appliance by SC_i, the appliance scheduling is conducted alternatively. If scheduling of appliance a_i is completed, the result will be stored in $\mathcal{X}_i$ where the $\mathcal{X}_i$ is set of scheduling result by each time slot t and this is described as follows:

$$\mathcal{X}_i \triangleq \left[x_i^1, x_i^2, x_i^3, \cdots, x_i^t\right], \tag{35}$$

where

$$x_i^t = \begin{cases} 1, & \text{if appliance } a_i \text{ is scheduled at } t \\ 0, & \text{otherwise.} \end{cases}$$

If the scheduling sequence is i, the scheduling parameter $\mathcal{S}_i$ and energy resource parameter g_i can be obtained with following formulation.

$$\operatorname*{argmin}_{\mathcal{S}_i, g_i} \sum_{t=1}^{T} \left\{ \frac{\lambda}{\Gamma_B} \cdot B_t \left(\sum_{j=0}^{i-1} g_j \cdot x_j^t \cdot E_j^{on} + g_i \cdot s_i^t \cdot E_i^{on} \right) + \frac{1-\lambda}{\Gamma_U} \cdot \sum_{a_i \in \mathcal{A}} \frac{w_i^t}{\rho_i} \right\} \tag{36}$$

subject to

$$\sum_{j=0}^{i-1} g_j \cdot x_j^t \cdot E_j^{on} + g_i \cdot s_i^t \cdot E_i^{on} \leq BO \tag{37}$$

$$0 \leq \lambda \leq 1 \tag{38}$$

$$w_i^t \leq W^* \cdot s_i^t \tag{39}$$

$$0 \leq w_i^t \tag{40}$$

$$\sum_{t}^{T} s_i^t = \rho_i \tag{41}$$

$$RER(t) = RER(0) + \int_{k=0}^{t-1} c(k) - \sum_{j=0}^{i} (1-g_j) \cdot x_j^t \cdot E_j^{on} \tag{42}$$

$$0 \leq RER(t) \leq RER_{max}, \forall t \in \mathcal{T}. \tag{43}$$

In summary, the heuristic algorithm for calculating approximated near-optimal solution is presented in Algorithm 1. Given the set of appliance profiles, the scheduling sequence is determined based on sensitivity of user convenience. After scheduling sequence has been set, iterative scheduling is conducted from the appliance with most sensitive user satisfaction. Before proceeding to scheduling the next appliance (a_{i+1}), the scheduling information X_i will be updated.

To assess the performance of the heuristic algorithm in terms of complexity, the computational complexity has been analyzed in each phase as follows:

1) For the phase of appliance sorting, an insertion sort algorithm is used in this paper and it requires n^2 comparisons for the sorting procedure. Therefore, the appliance sorting phase is quadratic with regard to the size of N, i.e., $O(n^2)$.
2) For the phase of appliance scheduling, the optimization objective function (36) is computed. The complexity of the procedure is linear with regard to the size of N. Therefore, the appliance scheduling phase is quadratic with regard to the size of N since it computes (36) for N times, i.e., complexity $O(n^2)$.

Finally, the entire algorithm takes the computational complexity as $O(n^2)$.

Algorithm 1: The Heuristic Algorithm

Step 1: Appliance Sorting
for $i = 0 \rightarrow N$ **do**
  Compute SC_i with (34)
  for $j = 0 \rightarrow i$ **do**
    if $SC_j > SC_i$ **then**
      SWAP SC_j and SC_i
    end if
  end for
end for
Step 2: Appliance Scheduling
for $j = 0 \rightarrow N$ **do**
  Schedule j-th appliance with (36), (37), (38), (39), (40), (41), (42), and (43).
  Update the X_j.
end for

TABLE II: Simulation Parameters

a_i	mean of E_i^{on} (w/h)	mean of ρ_i (time slot)
1	300	4
2	400	4
3	200	10
4	150	10
5	100	20

IV. Performance Evaluation

In this section, the performance of the proposed demand response scheme is evaluated. For the performance evaluation, the MOSEK optimization tool is used which is generally used and popular in the literature [14].

The simulation parameters are as follows. In order to model the charging of renewable energy resources, "*Real Italian Normalized PV Generation Profile*" is used [15]. To compute the quadratic cost function (7), assume that $\alpha_t = 5/9$, $\beta_t = 0$, and $\gamma_t = 0$ [9]. To demonstrate the performance of the proposed scheme, small scale topologies and large scale topology are configured. In the small scale topologies, the heuristic algorithm are verified that it can compute approximated near-optimal solutions. In the large scale topology, it is verified that the proposed scheme algorithm efficiently applies to large scale topologies for progressive and real time

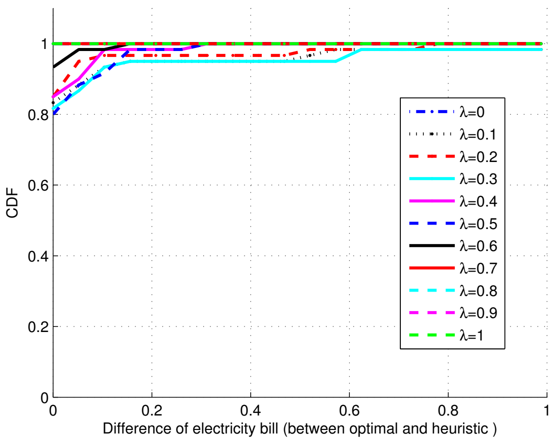

Fig. 3: CDF vs. Difference of electricity bill

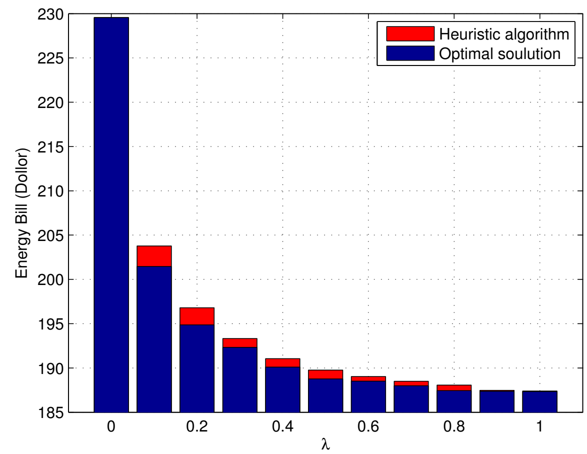

Fig. 4: Electricity bill vs. λ

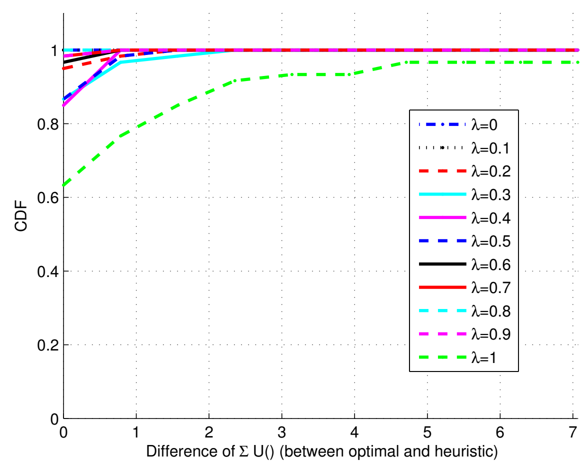

Fig. 5: CDF vs. Difference of $\sum \mathcal{U}$

pricing policies. Moreover, the existing scheme presented in [11] is compared with the proposed scheme in this intensive performance evaluation.

A. Approximation of the Proposed Scheme

Because the objective function is non-convex, it cannot achieve good performance for large scale topologies. Therefore, the heuristic algorithm is proposed as presented in Section III-B. To verify the proposed algorithm, a small scale appliance topology are generated in a random manner. Each appliance has 5 parameters as follows and the parameters are randomly generated. The first indicates the index of appliance (a_i). The second indicates the operating power (E_i^{on}) and follows a Poisson distribution. The third indicates the operating duration (ρ_i) and follows a Poisson distribution. The fourth and fifth parameters indicate the preferred start time (C_i^{begin}) and preferred end time (C_i^{end}) of appliance a_i, respectively. C_i^{begin} and C_i^{end} are generated by a uniform distribution. To approximately measure the proposed scheme, this simulation assumes that the smart grid system has five appliances ($N = 5$) and simulate for 60 days. For the simulation, parameters are as shown in the Table II. Since there are a few appliances, an optimal solution are computed using brute-force. Therefore, the heuristic algorithm is compared with the optimal solution with brute-force.

Fig. 3 shows the difference of electricity bill between the optimal solution and the heuristic algorithm. As shown in the figure, the heuristic algorithm guarantees more than 80% optimal for all λ. Moreover, more than 99% of the difference between the heuristic algorithm and optimal solution are less than 1$.

Fig. 4 shows the electricity bill graph according to λ. Both the results of heuristic algorithm and optimal solution show that electricity bills increase as λ decrease. This is because λ is more smaller, objective function considers the user convenience rather than electricity bill. If λ is 0.1, the maximum difference is about $2 and electricity bill is about $204. Therefore, it is obvious that the heuristic algorithm is more than 99% in the value of the optimal solution. Moreover, it can be observed that the heuristic algorithm works well regardless of λ.

Fig. 5 shows the difference of the sum of user convenience between the optimal solution and the heuristic algorithm. As shown in the figure, the heuristic algorithm guarantees more than 80% optimal for λ are 0 to 0.9. Moreover, the results of heuristic algorithm except $\lambda = 1$ show the approximate optimal solution less than 3. If λ is 1, however, there are a lot of the difference between heuristic algorithm and optimal solution. This is because that objective function tries to evenly distribute energy consumption to timeslots. (i.e., there are no rule time slot scheduling). However, it is not effect to the result of objective function since user convenience is multiplied to 0 if λ is 1. As in the figures, if λ is 0, the user satisfaction of heuristic algorithm are 100% consistent with optimal solution. If λ is 0, the objective function will only consider user satisfaction. Therefore, the results of appliance scheduling are their preferred times (i.e., $\sum \mathcal{U}(\cdot)$ will be 0). In the proposed heuristic algorithm, each appliance is alternatively scheduled. The first appliance is scheduled to satisfy its preferred time, and then the next appliance will be scheduled to satisfy its

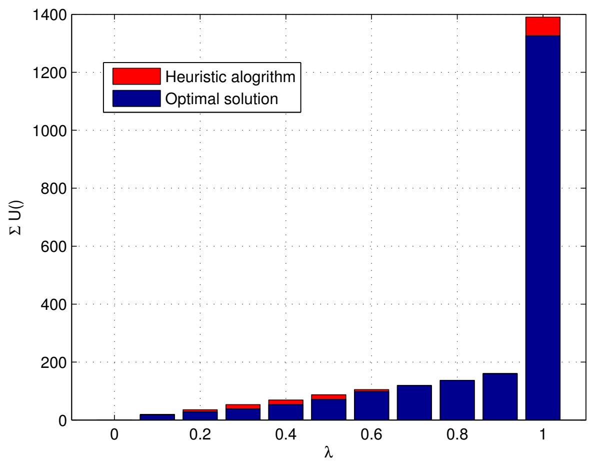

Fig. 6: $\sum \mathcal{U}$ vs. λ

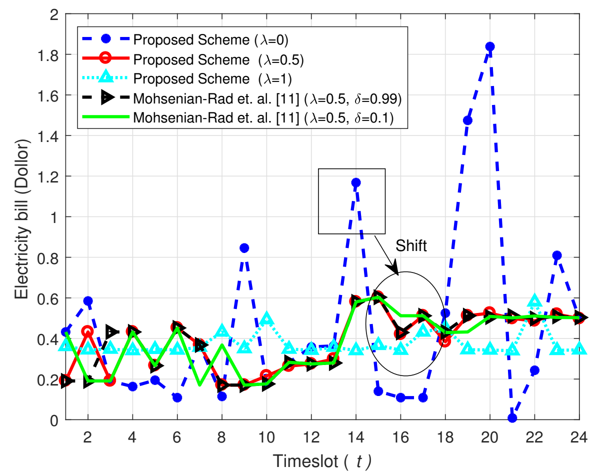

Fig. 7: Electricity bill vs. time slot (progressive policy)

preferred time. Therefore, the summation of user satisfaction will also be 0 in the heuristic algorithm.

Fig. 6 shows the user convenience graph according to λ. Both the user convenience of heuristic algorithm and optimal solution increase as λ increase. Therefore, it is observed that λ controls the user convenience efficiently. If λ is 0, both of the heuristic algorithm and optimal solution 100% guarantee user convenience. The sum of user convenience rapidly increase at λ is 1. Moreover, there are a lot of difference if λ is 1. The reason of that $\lambda = 1$ has abnormal result is the same reason as before. As shown in the figure, it can be observed that the sum of user convenience is more approximate than electricity bill except λ is 1. This is because of scheduling sequence is determined based on sensitivity of user convenience in the heuristic algorithm.

B. Efficiency of the Proposed Scheme for Large Topology

In this section, the application of the proposed scheme is simulated for the given large scale topology. To verify the proposed scheme, the appliance set ($N = 33$) is designed. Based on this topology, the performance of proposed algorithm and comparison scheme [11] is measured. In the simulation, the comparison schemes with $\delta = 0$ and $\delta = 1$ are evaluated. If $\delta = 0$, the comparison scheme only considers cost reduction. If $\delta = 0.99$, the comparison scheme considers the delay cost as well as electricity bill reduction. For a fair comparison, the delay tolerance is set to the end of the day and the objective function for the electricity bill follows (6), (7), and (8).

Fig. 7 shows the electricity bill graph according to timeslot when progressive policy is applied. As shown in the figure, electricity bill fluctuates by time slot if λ is zero (i.e., all appliance is scheduled at preferring time). On the other hand, electricity bill is relatively flat if λ is 1 (i.e., user convenience is not considered). If λ is 0.5, the graph is smoothly changed between graphs of $\lambda = 0$ and $\lambda = 1$. Therefore, it can be found that λ efficiently controls the electricity billing costs. In the figure, it can be found that appliance scheduling shifts from $t = 14$ to $t = 15 \sim 17$ to avoid peak load if λ is

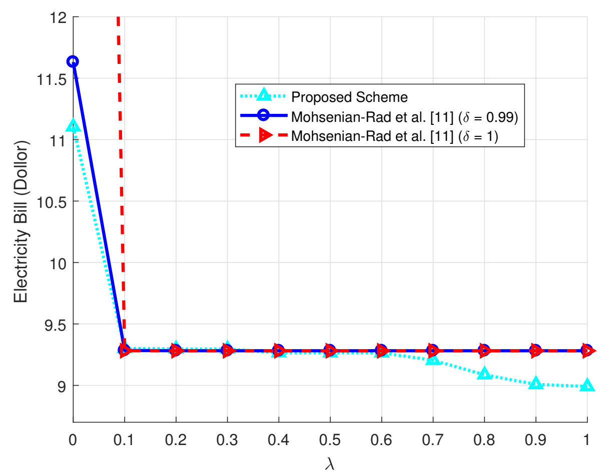

Fig. 8: Electricity bill vs. λ (progressive policy)

0.5. During $t = 15 \sim 17$, the bill of the proposed scheme is less than the bill of the comparison scheme. The comparison scheme shifts the peak load to a later timeslot. On the other hand, the proposed scheme can shift the load to an earlier or later timeslot.

Fig. 8 shows the sum of electricity bills according to λ when progressive policy is applied. As shown in the figure, electricity bill for the proposed scheme increases as λ decreases since objective function consider electricity bill as λ is greater. On the other hand, the bills for the comparison schemes almost did not change according to λ. It can be found that the user satisfaction model of the comparison scheme is not applicable to quadratic progressive policy. In addition, it can be seen that the electricity bills rapidly increase at $\lambda = 0$ for all schemes. Especially, the result of the comparison scheme with $\delta = 1$ is much greater than the results of the other schemes at $\lambda = 0$. The reason is that the scheduling can be concentrated to a single timeslot since $\lambda = 0$ induces the electricity bill increasing; and the comparison scheme with $\delta = 1$ does not consider user satisfaction (i.e., it randomly schedules

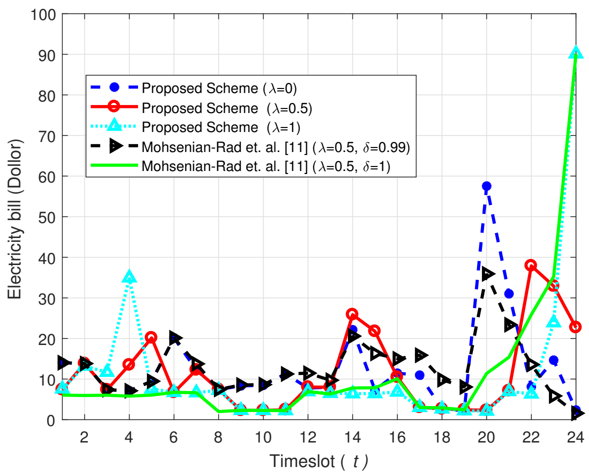

Fig. 9: Electricity bill vs. time slot (real time pricing policy)

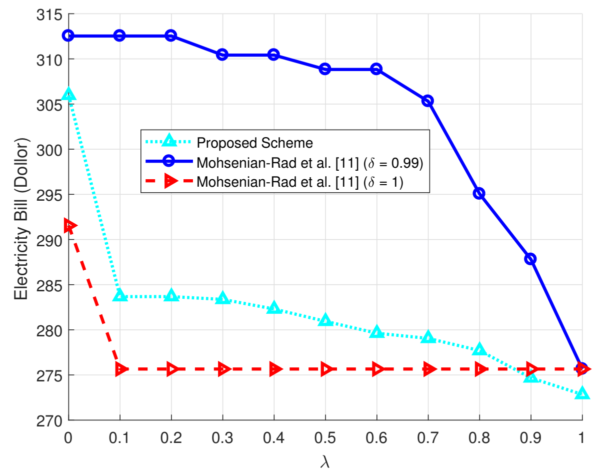

Fig. 10: Electricity bill vs. λ (real time pricing policy)

appliances). Interestingly, the bill of proposed scheme is less than the bill of the comparison scheme with $\delta = 0.99$. In the proposed scheme, user satisfaction degrees that belong to the preferred time are the same. In the comparison scheme with $\delta = 0.99$, however, the earlier timeslot has a greater user satisfaction although the timeslot belongs to the preferred time. Moreover, the bill of the proposed scheme is less than the bill of the comparison scheme with $\lambda = 1$. This is because the proposed scheme can shift the load to earlier or later timeslot as shown in the previous figure.

Fig. 9 shows the electricity bill graph according to timeslot when real time pricing policy is applied. As shown in the figure, electricity bill fluctuates by time slot if λ are 0 or 1. The result is difference with progressive policy if λ is 1. In the figure, electricity bill rapidly increases at timeslot 24. This is because appliance scheduling is concentrated at low electricity pricing time. If λ is 0.5, the graph is smoothly changed as graphs of $\lambda = 0$. Therefore, it can be found that λ also efficiently controls the electricity bill when real time pricing policy is applied. For the comparison scheme with $\delta = 1$, appliance scheduling is concentrated at timeslot 24 since $\delta = 1$ only considers electricity bill reduction. In contrast with the comparison scheme with $\delta = 0.99$, the scheduling of the proposed scheme is somewhat concentrated at timeslot 4. As shown in Fig. 2, $t = 23 \sim 24$ is the first low price period and $t = 4$ is the second low price period for all timeslots. If the number of request timeslots is greater than the number of first period timeslots and the begin time is later than timeslot 4, the proposed scheme will schedule at timeslot 4. However, the comparison scheme cannot schedule to an earlier timeslot.

Fig. 10 shows the sum of electricity bills according to λ when real time pricing policy is applied. As shown in the figure, electricity bill increases as λ decreases since objective function consider electricity bill as λ is greater. If real time policing is applied, it can be observed that the changing of electricity according to λ is less abrupt than progressive policy. This is because, progressive policy is represented by quadratic function. In contrast with the progressive policy, the comparison scheme with $\delta = 0.99$ efficiently controls λ in the real time pricing policy. The comparison scheme with $\delta = 1$ has the same results for the electricity bill from $\lambda = 0.1$ to $\lambda = 1$ since it only considers electricity bill reduction. At $\lambda = 0$, the reason for the different results among the three schemes is the same as the reason for the progressive results. If they only consider user satisfaction and the period of the preferred time is greater than the operation time, the electricity bill will randomly be different. At $\lambda = 1$, the bill for the proposed algorithm is less than the bill for the comparison scheme since the proposed scheme can schedule at timeslot 4, as seen in the previous figure.

V. Concluding Remarks

This paper proposes optimization formulations and corresponding algorithms for residential smart grid demand response systems for renewable energy resources. For the formulation, the electricity, renewable energy resources, and user convenience models are designed in order to satisfy demand response management for various electricity bill policies. In real time pricing formulation, non-convex programming is originally used and reformulation framework is proposed for converting the given non-convex to convex which can guarantee optimal solutions. Furthermore, the additional heuristic algorithm for progressive pricing formulation is designed for obtaining approximated near-optimal solutions. Via intensive simulations with the customized simulation software based on MOSEK optimization tool, it has been shown that the proposed heuristic algorithm presents near-optimal performance that is close to the optimal solutions. The proposed algorithm assumes that the operation and requests of the appliances are perfectly known in advance. Future research will focus on an appliance scheduling based on appliance usage patterns using machine learning algorithms.

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